

# Chapter 2

## Understanding the Analytic Hierarchy Process

In this chapter, we will explain the fundamentals of the Analytic Hierarchy Process. The reader is referred to the original Saaty's (2012) discussion of AHP or to Brunnelli's (2015) for a theoretical introduction to the method. In this book, AHP concepts will be explained from a practical point of view using examples for greater clarity.

To explain this method we will use a simple example<sup>1</sup>: Our goal is to purchase a new car. Our purchase is based on different criteria such as cost, comfort, and safety (the reader can think of many more but we will use only three for illustration purposes). We could evaluate several alternatives but let us assume that we have only two: Car 1 and Car 2. To analyze the decision of purchasing a car using the analytic hierarchy process we should follow the next steps:

- (1) Develop a model for the decision: Break down the decision into a hierarchy of goals, criteria, and alternatives.
- (2) Derive priorities (weights) for the criteria: The importance of criteria are compared pairwise with respect to the desired goal to derive their weights. We then check the consistency of judgments; that is, a review of the judgments is done in order to ensure a reasonable level of consistency in terms of proportionality and transitivity.

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Developing a Model <https://mix.office.com/watch/17icbrmswidq0>.

Deriving Priorities (weights) for the Criteria <https://mix.office.com/watch/4odxenri07nm>.

Deriving Local Priorities (preferences) for the Alternatives <https://mix.office.com/watch/1idaxl30c6o5o>.

Deriving Overall Priorities <https://mix.office.com/watch/ztkx3ea8lki8>.

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<sup>1</sup>For this chapter, it is recommended that the reader follows the calculations of this example using a spreadsheet.

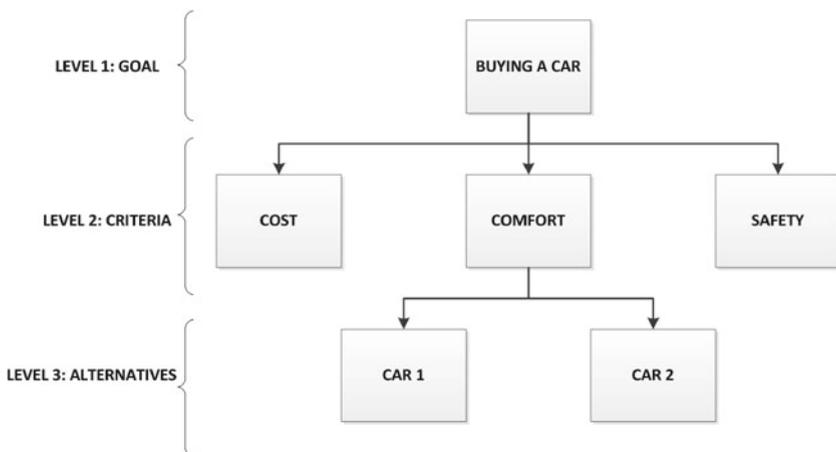
- (3) Derive local priorities (preferences) for the alternatives: Derive priorities or the alternatives with respect to each criterion separately (following a similar process as in the previous step, i.e., compare the alternatives pairwise with respect to each criterion). Check and adjust the consistency as required.
- (4) Derive Overall Priorities (Model Synthesis): All alternative priorities obtained are combined as a weighted sum—to take into account the weight of each criterion—to establish the overall priorities of the alternatives. The alternative with the highest overall priority constitutes the best choice.
- (5) Perform Sensitivity analysis: A study of how changes in the weights of the criteria could affect the result is done to understand the rationale behind the obtained results.
- (6) Making a Final Decision: Based on the synthesis results and sensitivity analysis, a decision can be made.

At this point, the reader may feel a little intimidated by terms such as judgments, priorities, pairwise comparison, consistency, etc.; however, the following discussion will clarify these topics.

## 2.1 Developing a Model

The first step in an AHP analysis is to build a hierarchy for the decision. This is also called decision modeling and it simply consists of building a hierarchy to analyze the decision.

The analytic hierarchy process (AHP) structures the problem as a hierarchy. Figure 2.1 shows the hierarchy proposed for our example. Note that the first level of



**Fig. 2.1** Decision hierarchy for buying a car

**Table 2.1** Saaty's pairwise comparison scale

Verbal judgment	Numeric value
Extremely important	9
	8
Very Strongly more important	7
	6
Strongly more important	5
	4
Moderately more important	3
	2
Equally important	1

the hierarchy is our goal; in our example, buying a car. The second level in the hierarchy is constituted by the criteria we will use to decide the purchase. In our example, we have mentioned three criteria: cost, comfort, and safety. The third level consists of the available alternatives.<sup>2</sup> In this case: Car 1 and Car 2.

The advantages of this hierarchical decomposition are clear. By structuring the problem in this way it is possible to better understand the decision to be achieved, the criteria to be used and the alternatives to be evaluated. This step is crucial and this is where, in more complex problems, it is possible to request the participation of experts to ensure that all criteria and possible alternatives have been considered. Also note that in complex problems it may be necessary to include additional levels in the hierarchy such as sub-criteria.

## 2.2 Deriving Priorities (Weights) for the Criteria

Not all the criteria will have the same importance. Therefore, the second step in the AHP process is to derive the relative priorities (weights) for the criteria. It is called relative because the obtained criteria priorities are measured with respect to each other as we will see in the following discussion.

It is clear that when buying a car (as in other decisions), not all criteria are equally important in a given time. For example, a student may give more importance to the cost factor rather than to comfort and safety, while a parent may give more importance to the safety factor rather than to the others. Clearly, the importance or weight of each criterion will be different and because of this, we first are required to derive by pairwise comparisons the relative priority of each criterion with respect to each of the others using a numerical scale for comparison developed by Saaty (2012) as shown in Table 2.1.<sup>3</sup>

<sup>2</sup>Each criterion, alternative and the goal are collectively referred as model elements.

<sup>3</sup>In this figure, the intermediate values 2, 4, 6 and 8 are used to address situations of uncertainty. For example, when the decision maker is in doubt whether to rate a pairwise comparison as

**Table 2.2** Pairwise comparison matrix of criteria for buying a car

Buying a car	Cost	Comfort	Safety
Cost			
Comfort			
Safety			

To perform the pairwise comparison you need to create a comparison matrix of the criteria involved in the decision, as shown in Table 2.2.

Cells in comparison matrices will have a value from the numeric scale shown in Table 2.1 to reflect our relative preference (also called intensity judgment or simply judgment) in each of the compared pairs. For example, if we consider that cost is very *strongly more important* than the comfort factor, the cost-comfort comparison cell (i.e., the intersection of the row ‘cost’ and column ‘comfort’) will contain the value 7 as shown in Table 2.3. Mathematically this means that the ratio of the importance of cost versus the importance of comfort is seven ( $\text{cost}/\text{comfort} = 7$ ). Because of this, the opposite comparison, the importance of comfort relative to the importance of cost, will yield the reciprocal of this value ( $\text{comfort}/\text{cost} = 1/7$ ) as shown in the comfort-cost cell in the comparison matrix in Table 2.3. The rationale is intuitively obvious. For example, if in daily life we say that an apple A is twice as big as apple B ( $A/B = 2$ ), this implies that apple B is half the size of apple A ( $B/A = 1/2$ ). Similarly, if we consider that cost is *moderately more important* than safety ( $\text{cost}/\text{safety} = 3$ ), we will enter 3 in the cost-safety cell (using the scale from Table 2.1) and the safety-cost cell will contain the reciprocal  $1/3$  ( $\text{safety}/\text{cost} = 1/3$ ). Finally, if we feel that safety is *moderately more important* than comfort, the safety-comfort cell will contain the value 3 and the comfort-safety cell, will have the reciprocal  $1/3$ . Once all these judgments are entered in the pairwise comparison matrix (Table 2.2) we obtain the results shown in Table 2.3.

Note in the comparison matrix of Table 2.3 that when the importance of a criterion is compared with itself; for example, cost versus cost, comfort versus comfort, or safety versus safety; the input value is 1 which corresponds to the intensity of equal importance in the scale of Table 2.1. This is intuitively sound because the ratio of the importance of a given criterion with respect to the importance of itself will always be equal.

At this stage, you can see one of the great advantages of the analytic hierarchy process: its natural simplicity. Regardless of how many factors are involved in making the decision, the AHP method only requires to compare a pair of elements at any time; something that, because of our pair anatomy (e.g., two hands), we have done for centuries. Another important advantage is that it allows the inclusion of tangible variables (e.g., cost) as well as intangible ones (e.g., comfort) as criteria in the decision. The comparison matrix (Table 2.3) shows the pairwise relative

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(Footnote 3 continued)

“moderately more important (3)” or “strongly more important (5)”, a possible solution is to rate it as “From moderately to strongly more important;” that is, a 4.

**Table 2.3** Pairwise comparison matrix with intensity judgments

Buying a car	Cost	Comfort	Safety
Cost	1	7	3
Comfort	1/7	1	1/3
Safety	1/3	3	1

**Table 2.4** Column addition

Buying a car	Cost	Comfort	Safety
Cost	1.000	7.000	3.000
Comfort	0.143	1.000	0.333
Safety	0.333	3.000	1.000
Sum	1.476	11.000	4.333

**Table 2.5** Normalized matrix

Buying a car	Cost	Comfort	Safety
Cost	0.677	0.636	0.692
Comfort	0.097	0.091	0.077
Safety	0.226	0.273	0.231

priorities for the criteria. We now need to calculate the overall priorities or weights of the criteria. There are two methods available for this purpose: the exact and the approximate.

Although we will not show the *exact method* in detail here, the general idea is very simple. Raise the comparison matrix to powers (e.g., raise the matrix to the power of two, raise the resulting matrix to the power of two again, and so forth) a few times until all the columns become identical. This is called the limit matrix. At this point, any of the matrix columns constitutes the desired set of priorities. This calculation can be done in a spreadsheet but it is currently done very easily using AHP-based software packages.

As we aim to explain roughly the elements of the AHP method, we will rather use the *approximate method* in our example due to its simplicity. However, keep in mind that this method provides a valid approximation to the overall weights only when the comparison matrix has a very low inconsistency.<sup>4</sup>

The *approximate method* requires the normalization of the comparison matrix; i.e., add the values in each column (Table 2.4).

Next, divide each cell by the total of the column (Table 2.5). The normalized matrix is shown in Table 2.5.

From this normalized matrix, we obtain the overall or final priorities (Table 2.8) by simply calculating the average value of each row (e.g., for the cost row:  $0.677 + 0.636 + 0.692/3 = 0.669$ ).

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<sup>4</sup>Inconsistency will be explained later in our discussion.

**Table 2.6** Calculation of priorities: row averages

Buying a car	Cost	Comfort	Safety	Priority
Cost	0.677	0.636	0.692	<b>0.669</b>
Comfort	0.097	0.091	0.077	<b>0.088</b>
Safety	0.226	0.273	0.231	<b>0.243</b>

**Table 2.7** Presentation of results: original judgments and priorities

Buying a car	Cost	Comfort	Safety	Priority
Cost	1.000	7.000	3.000	<b>0.669</b>
Comfort	0.143	1.000	0.333	<b>0.088</b>
Safety	0.333	3.000	1.000	<b>0.243</b>

Although there is no standardized way of presenting the results, showing the comparison matrix with the original judgments (Table 2.4) along with the calculated priorities (obtained in Table 2.6) is a useful way to see the judgments and priorities at the same time, as it can be seen in Table 2.7.

According to the results in Table 2.7, it is clear that—for this example—we give more importance to the cost criterion (0.669), followed by safety (0.243). The comfort factor has a minimum weight (0.088) in our purchasing decision. Another important observation is that the pairwise comparison of criteria, through questions such as: to purchase your car, what is more important: cost or comfort?, allows us to derive, based on our preferences, the final priorities or weights for the criteria. That is, the priorities are not assigned arbitrarily but are derived based on our judgments and preferences. These priorities have mathematical validity, as measurement values derived from a ratio scale, and have also an intuitive interpretation. From Table 2.7 we can interpret that cost has 66.9 % of the overall importance of the criteria, followed by safety with 24.3 % and comfort (8.8 %), respectively.

### 2.3 Consistency

Once judgments have been entered, it is necessary to check that they are consistent. The idea of consistency is best illustrated in the following example: If you prefer an apple twice as much than a pear and a pear twice as much than an orange; how much would you prefer an apple with respect to an orange? The mathematically consistent answer is 4. Similarly, in a comparison matrix criteria, if we provide a value of 2 to the first criterion over the second and assign a value of 3 to the second criterion with respect to the third one, the value of preference of the first criterion with respect to the third one should be  $2 \times 3 = 6$ . However, if the decision-maker has assigned a value such as 4, 5, or 7, there would be a certain level of inconsistency in the matrix of judgments. Some inconsistency is expected and allowed in AHP analysis.

Since the numeric values are derived from the subjective preferences of individuals, it is impossible to avoid some inconsistencies in the final matrix of judgments. The question is how much inconsistency is acceptable. For this purpose, AHP calculates a consistency ratio (CR) comparing the consistency index (CI) of the matrix in question (the one with our judgments) versus the consistency index of a random-like matrix (RI). A random matrix is one where the judgments have been entered randomly and therefore it is expected to be highly inconsistent. More specifically, RI is the average CI of 500 randomly filled in matrices. Saaty (2012) provides the calculated RI value for matrices of different sizes as shown in Table 2.8.

In AHP, the consistency ratio is defined as  $CR = CI/RI$ . Saaty (2012) has shown that a consistency ratio (CR) of 0.10 or less is acceptable to continue the AHP analysis. If the consistency ratio is greater than 0.10, it is necessary to revise the judgments to locate the cause of the inconsistency and correct it.

Since the calculation of the consistency ratio is easily performed by computer programs, we limit ourselves here to producing an estimate of this value as follows:

- (a) Start with the matrix showing the judgment comparisons and derived priorities (Table 2.7) which is reprinted for convenience in Table 2.9.
- (b) Use the priorities as factors (weights) for each column as shown in Table 2.10.
- (c) Multiply each value in the first column of the comparison matrix in Table 2.10 by the first criterion priority (i.e.,  $1.000 \times 0.669 = 0.669$ ;  $0.143 \times 0.669 = 0.096$ ;  $0.333 \times 0.669 = 0.223$ ) as shown in the first column of Table 2.11; multiply each value in the second column of the second

**Table 2.8** Consistency indices for a randomly generated matrix

n	3	4	5	6
RI	0.58	0.9	1.12	1.24

**Table 2.9** Prioritization results

Buying a car	Cost	Comfort	Safety	Priority
Cost	1.000	7.000	3.000	0.669
Comfort	0.143	1.000	0.333	0.088
Safety	0.333	3.000	1.000	0.243

**Table 2.10** Priorities as factors

Buying a car	Cost	Comfort	Safety
<i>Criteria Weights -&gt;</i>	0.669	0.088	0.243
Cost	1.000	7.000	3.000
Comfort	0.143	1.000	0.333
Safety	0.333	3.000	1.000

**Table 2.11** Calculation of weighted columns

Buying a car	Cost	Comfort	Safety
Cost	0.669	0.617	0.729
Comfort	0.096	0.088	0.081
Safety	0.223	0.265	0.243

**Table 2.12** Calculation of weighted sum

Buying a car	Cost	Comfort	Safety	Weighted sum
Cost	0.669	0.617	0.729	2.015
Comfort	0.096	0.088	0.081	0.265
Safety	0.223	0.265	0.243	0.731

**Table 2.13** Calculation of  $\lambda_{\max}$ 

Weighted sum	Priority	
2.015/	0.669 =	3.014
0.265/	0.088 =	3.002
0.731/	0.243 =	3.005
	Total	9.021
	Divide Total by 3 to obtain $\lambda_{\max}$ =	3.007

criterion priority; continue this process for all the columns of the comparison matrix (in our example, we have three columns). Table 2.11 shows the resulting matrix after this process has been completed.

- (d) Add the values in each row to obtain a set of values called weighted sum as shown in Table 2.12.
- (e) Divide the elements of the weighted sum vector (obtained in the previous step) by the corresponding priority of each criterion as shown in Table 2.13. Calculate the average of the values from the previous step; this value is called  $\lambda_{\max}$ .

$$\lambda_{\max} = (3.014 + 3.002 + 3.005)/3 = 3.007.$$

- (f) Now we need to calculate the consistency index (CI) as follows:

$$\text{C.I.} = (\lambda_{\max} - n)/(n - 1)$$

where n is the number of compared elements (in our example n = 3).

Therefore,

$$CI = (\lambda_{\max} - n)/(n - 1) = (3.007 - 3)/(3 - 1) = \mathbf{0.004}$$

(g) Now we can calculate the consistency ratio, defined as:

$$CR = CI/RI$$

Therefore,

$$CR = CI/RI = 0004/0.58 = \mathbf{0.006}$$

CI is the consistency index calculated in the previous step with a value of 0.004. RI is the consistency index of a randomly generated comparison matrix and is available to the public in tables (Table 2.8). In other words, RI is the consistency index that would be obtained if the assigned judgment values were totally random. It is possible to show (this is beyond the scope of this book) that the value of RI depends on the number of items (n) that are being compared (see expected values shown in Table 2.8). It can be seen that for n = 3, RI = 0.58. Using these values for CI and RI, it can be calculated that

$$CR = 0.004/0.58 = 0.006$$

Since this value of 0.006 for the proportion of inconsistency CR is less than 0.10, we can assume that our judgments matrix is reasonably consistent so we may continue the process of decision-making using AHP.

## 2.4 Deriving Local Priorities (Preferences) for the Alternatives

Our third step consists of deriving the relative priorities (preferences) of the alternatives with respect to each criterion. In other words, what are the priorities of the alternatives with respect to cost, comfort, and safety respectively? Since these priorities are valid only with respect to each specific criterion, they are called local priorities to differentiate them from the overall priorities to be calculated later.

As indicated, we need to determine the priorities of the alternatives with respect to each of the criteria. For this purpose, we do a pairwise comparison (using the numeric scale from Table 2.1) of all the alternatives, with respect to each criterion, included in the decision making model. In a model with two alternatives it is required to make only one comparison (Alternative 1 with Alternative 2) for each criterion; a model with three alternatives would require to make three comparisons (Alternative 1 with Alternative 2, Alternative 2 with Alternative 3, and Alternative 1

with alternative 3) for each criterion; and so on. There will be as many alternative comparison matrices as there are criteria.

In our example, we only have two alternatives: Car 1 and Car 2 and we have three criteria. This means that there will be three comparison matrices corresponding to the following three comparisons:

With respect to the cost criterion: Compare Car 1 with Car 2

With respect to the comfort criterion: Compare Car 1 with Car 2

With respect to the safety criterion: Compare Car 1 with Car 2.

We can do these comparisons through a series of questions as shown below with sample answers.

**Comparison Question 1:** With respect to the cost criterion, which alternative is preferable: Car 1 or Car 2?

For our example, let us assume that we prefer *very strongly* (using the scale in Table 2.1) the Car 1 over the Car 2. This means that in the Car 1–Car 2 cell (i.e., the cell intersected by the row “Car 1” and the column “Car 2”) of our comparison matrix regarding cost alternatives (Table 2.14), we assign a value of 7 (value assigned using the scale from Table 2.1) to reflect our preference. Similarly, we assign the reciprocal reverse 1/7 in the Car 2–Car 1 cell in the table.

By normalizing the matrix and averaging the rows we obtain the priorities (or preferences) for each of the alternatives (Table 2.15) with respect to cost.

Because these priorities apply only to the cost criterion, they are called *local priorities* with respect to cost. The results are summarized for convenience as shown in Table 2.16.

**Comparison Question 2:** With respect to the comfort criterion, which alternative is preferable: Car 1 or Car 2?

Assume that Car 2 is *strongly preferred* over the Car 1; that is, we assign a value of 5 (using scale from Table 2.1) in the cell Car 2–Car 1 in our comparison matrix

**Table 2.14** Comparison with respect to cost

Cost	Car 1	Car 2
Car 1	1.000	7.000
Car 2	0.143	1.000
Sum	1.143	8.000

**Table 2.15** Preference with respect to cost

Cost	Car 1	Car 2	Priority
Car 1	0.875	0.875	0.0875
Car 2	0.125	0.125	0.125

**Table 2.16** Results with respect to cost

Cost	Car 1	Car 2	Priority
Car 1	1.000	7.000	0.875
Car 2	0.143	1.000	0.125

regarding comfort alternatives and the reciprocal reverse 1/5 in the Car 1–Car 2 cell (see Table 2.17).

By normalizing the matrix and averaging the rows we obtain the local priorities (or preferences) for each one of the alternatives (Table 2.17) with respect to comfort. See Table 2.18.

The results are summarized in Table 2.19.

**Comparison Question 3:** With respect to the safety criterion, which alternative is preferable: Car 1 or Car 2?

For our example, let us say the Car 2 is extremely preferable to Car 1 with respect to this criterion. These judgments are entered numerically (using scale from Table 2.1) in the respective cells in Table 2.20.

By normalizing the matrix and averaging the rows we obtain the local priorities (or preferences) for each one of the alternatives (Table 2.21) with regard to safety.

The results are summarized in Table 2.22.

Notice that having only two alternatives to compare with respect to each criterion, simplifies the calculations with respect to consistency. When there are only

**Table 2.17** Comparison with respect to comfort

Comfort	Car 1	Car 2
Car 1	1.000	0.200
Car 2	5.000	1.000
Sum	6.000	1.200

**Table 2.18** Preference with respect to comfort

Comfort	Car 1	Car 2	Priority
Car 1	0.167	0.167	0.167
Car 2	0.833	0.833	0.833

**Table 2.19** Results with respect to comfort

Comfort	Car 1	Car 2	Priority
Car 1	1.000	0.200	0.167
Car 2	5.000	1.000	0.833

**Table 2.20** Comparison with respect to safety

Safety	Car 1	Car 2
Car 1	1.000	0.111
Car 2	9.000	1.000
Sum	10.000	1.111

**Table 2.21** Preferences with respect to safety

Safety	Car 1	Car 2	Priority
Car 1	0.100	0.100	0.100
Car 2	0.900	0.900	0.900

**Table 2.22** Results with respect to safety

Safety	Car 1	Car 2	Priority
Car 1	1.000	0.111	0.100
Car 2	9.000	1.000	0.900

**Table 2.23** Local Priorities (or preferences) of the alternatives with respect to each criterion

Alternatives	Cost	Comfort	Safety
Car 1	0.875	0.167	0.100
Car 2	0.125	0.833	0.900

two elements to compare (in our example, Car 1 and Car 2), the respective comparison matrices (Tables 2.14, 2.17 and 2.20) will always be consistent ( $CR = 0$ ). However, consistency must be checked if the number of elements pairwise compared is 3 or more.

We can summarize the results of this step by indicating that if our only criterion were cost, Car 1 would be our best option (priority = 0.875 in Table 2.16); if our only criterion were comfort, our best bet would be the Car 2 (0.833 priority in Table 2.19) and finally, if our sole purchasing criteria were safety, our best option would be the Car 2 (0.900 priority in Table 2.22). As previously indicated, the priorities (preferences) of the alternatives, with respect to each criterion, are called local priorities (or preferences). The summary of the local priorities for each alternative is shown in Table 2.23.

## 2.5 Derive Overall Priorities (Model Synthesis)

Up to this point we have obtained local priorities which indicate the preferred alternative with respect to each criterion. In this fourth step, we need to calculate the overall priority (also called final priority)<sup>5</sup> for each alternative; that is, priorities that take into account not only our preference of alternatives for each criterion but also the fact that each criterion has a different weight. Given that we are using all the values provided in the model, this step is called model synthesis.

We start the calculation of the overall priority using the local priority of each alternative as the starting point (Table 2.23, also repeated for convenience as Table 2.24).

Next, we need to take into consideration the weights of each criteria (from 2.8) and for this purpose they are inserted in the table as shown in Table 2.25.

For example, the cost criterion has a priority (or weight) of 0.669 and the Car 1 has a local priority (or preference) of 0.875 relative to cost; therefore, the weighted

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<sup>5</sup>It is customary to refer to overall (also called general or final) priorities of the alternatives when they are calculated with respect to the whole model; that is, after the synthesis process.

**Table 2.24** Local priorities table as a base

	Cost	Comfort	Safety
Car 1	0.875	0.167	0.100
Car 2	0.125	0.833	0.900

**Table 2.25** Preparation for weighing of priorities

	Cost	Comfort	Safety
<i>Criteria Weights -&gt;</i>	<i>0.669</i>	<i>0.088</i>	<i>0.243</i>
Car 1	0.875	0.167	0.100
Car 2	0.125	0.833	0.900

**Table 2.26** Calculation of overall priorities

	Cost	Comfort	Safety	Overall priority
<i>Criteria Weights -&gt;</i>	<i>0.669</i>	<i>0.088</i>	<i>0.243</i>	
Car 1	0.585	0.015	0.024	0.624
Car 2	0.084	0.074	0.219	0.376

**Table 2.27** Synthesis of the model

	Cost	Comfort	Safety	Overall priority
<i>Criteria Weights -&gt;</i>	<i>0.669</i>	<i>0.088</i>	<i>0.243</i>	
Car 1	0.875	0.167	0.100	0.624
Car 2	0.125	0.833	0.900	0.376

priority, with respect to cost, of the Car 1 is:  $0.669 \times 0.875 = 0.585$ . A similar calculation is necessary to obtain the Car 1 weighted priorities with respect to comfort and safety criteria. The resulting matrix is shown in Table 2.26. Finally, the overall priority of the Car 1 is obtained by adding these results along the row. This procedure is repeated for each of the alternatives being evaluated. The overall priorities of the alternatives are shown in the rightmost column of Table 2.26.

The calculations for each alternative are shown below and the results are presented in Table 2.27 following the convention of showing the local priorities (cells) and the weights for each criterion (at the top of each column). This process is called the model synthesis (see Table 2.27).

In other words

$$\text{Overall Priority of the Car 1: } 0.875 \times 0.669 + 0.167 \times 0.088 + 0.100 \times 0.243 = 0.624$$

$$\text{Overall Priority of the Car 2: } 0.125 \times 0.669 + 0.833 \times 0.088 + 0.900 \times 0.243 = 0.376$$

Now we can list the alternatives ordered by their overall priority or preference as follows:

Alternatives	Overall Priority
1. <i>Car 1</i>	<b>0.624</b>
2. <i>Car 2</i>	0.376

In other words, given the importance (or weight) of each buying criteria (cost, comfort, and safety), the Car 1 is preferable (overall priority = 0.624) compared to the Car 2 (overall priority = 0.376).

## 2.6 Sensitivity Analysis

The overall priorities will be heavily influenced by the weights given to the respective criteria. It is useful to perform a “what-if” analysis to see how the final results would have changed if the weights of the criteria would have been different. This process is called sensitivity analysis and constitutes the fifth step in our AHP methodology. Sensitivity analysis allows us to understand how robust is our original decision and what are the drivers (i.e., which criteria influenced the original results). This is an important part of the process and, in general, no final decision should be made without performing sensitivity analysis.

Note that in our example (Table 2.27), the cost is of great importance (priority 0.669) and given that the Car 1 has a high local priority (0.875) for this single criterion, undoubtedly this influences the final result favorably for the Car 1. The questions that we can ask ourselves at this stage are: What would be the best alternative if we change the importance of the criteria? What if we give the same importance to all the criteria? And, what if we give more importance to safety or we consider it to be as important as the cost? and so on.

To perform a sensitivity analysis, it is necessary to make changes to the weights of the criterion and see how they change the overall priorities of the alternatives. To exemplify this we will analyze the following scenarios: (a) when all the criteria have the same weight and (b) what weight is needed for the cost criterion to lead to a tie in the overall priorities of the alternatives? (This is a logical question since we know cost is very important in the original analysis and Car 1 scores very high in

**Table 2.28** Original scenario—synthesis of the model

	Cost	Comfort	Safety	Overall priority
<i>Criteria Weights -&gt;</i>	<i>0.669</i>	<i>0.088</i>	<i>0.243</i>	
Car 1	0.875	0.167	0.100	0.624
Car 2	0.125	0.833	0.900	0.376

**Table 2.29** Scenario—all criteria have the same weight

	Cost	Comfort	Safety	Overall priority
<i>Criteria Weights -&gt;</i>	<i>0.333</i>	<i>0.333</i>	<i>0.333</i>	
Car 1	0.875	0.167	0.100	0.380
Car 2	0.125	0.833	0.900	0.619

**Table 2.30** Third scenario: cost weight leading to equal preferences of the alternatives

	Cost	Comfort	Safety	Overall priority
<i>Criteria Weights</i>	<i>0.500</i>	<i>0.250</i>	<i>0.250</i>	
Car 1	0.875	0.167	0.100	0.504
Car 2	0.125	0.833	0.900	0.496

this criterion causing Car 1 to be the best preference). Table 2.28 is the original model synthesis (Table 2.27) where the most preferable option is the Car 1 (0.624).

Table 2.29 shows the case where all 3 criteria have the same weight (0.333). In this second scenario, the final choice no longer has the Car 1 but the Car 2 (0.619) as the best option. This is because the Car 2 wins in all criteria except for cost. By lowering the weight of cost (from 0.669 in the original scenario to 0.333 in the second stage), its cost disadvantage is not as noticeable.

This also suggests that both alternatives are equally preferred when cost weighs in the range of 0.333–0.669. To calculate the break-even point we can try different weights for the cost and find that when the cost weighs approximately 0.5 of the overall criterion importance, the Car 1 and the Car 2 have the same preference for practical purposes. That is, both alternatives are equally preferred as shown in Table 2.30.

## 2.7 Making a Final Decision

Once the above steps have been completed, it is now possible to make a decision. This constitutes the last step in our AHP analysis. For this, it is necessary to compare the overall priorities obtained and whether the differences are large enough to make a clear choice. It is also necessary to analyze the results of the sensitivity analysis (Tables 2.28, 2.29 and 2.30). From this analysis, we can express our final recommendation as follows: If the importance of the cost is more than 50 % of the overall importance of the criteria in the decision, the best alternative is the Car 1 (Table 2.28); however, if the importance of cost is much less than 50 %, the Car 2 is the best decision (From Tables 2.29 and 2.30).

## 2.8 Conclusion

It is important to note that, contrary to the common belief, the system does not determine the decision we should make, rather, the results should be interpreted as a blueprint of preference and alternatives based on the level of importance obtained for the different criteria taking into consideration our comparative judgments. In other words, the AHP methodology allows us to determine which alternative is the most consistent with our criteria and the level of importance that we give them.

Although AHP calculations can be done using electronic spreadsheets, the appearance of software packages such as Expert Choice (2015) in the late eighties and Super Decisions (2015) and Decision Lens (2015) later on has made AHP mathematical calculations very easy to deal with. A survey of current AHP software packages is beyond the scope of this book. However, the reader is referred to Ishizaka and Nemery (2013) for a partial survey of AHP software packages.

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